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A Segmented Market Model Incorporating Endogenous Policy Risk

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This paper develops a segmented market model to incorporate policy risk. Credit risks and information inertia are considered within the model's framework. The model shows that a high level of policy uncertainty has a negative impact on the capital market's participation rate, which in turn, determines the effectiveness of monetary policy. This suggests that policy risk is an endogenous factor, and that the authority should control policy uncertainties.

I. Introduction

In a standard classical model without market segmentation, persistent money injections increase expected inflation, leaving no effects on real interest rates. According to the Fisher equation, nominal interest rate is supposed to rise as open market operations increase money supply. This strand of Models had failed to account for the liquidity effect evidenced by empirical observations: the policy-induced extra money in the economy pushes down interest rates, thereby stimulating economic activities. The liquidity effect provides some validation to the New Keynesian doctrine, Clarida et al. (1999)'s classical piece illustrates how macroeconomic stabilization efforts by monetary authority foster efficient macroeconomic outcomes.

The segmented market model, which assumes that markets are incomplete, reconciles the liquidity effect and the quantity theory of money within the neoclassical framework. Under such neoclassical models, only a fraction λ of agents, are actively trading in any given period. The asset market is thus incomplete or segmented. Here, the financial market participation rate is defined by λ. The concept of financial market participation is approached differently in the Appendix, offering a more pragmatic angle.

When the government injects money through open market operations, only active agents are on the other side of the transactions. If active agents expect the expansionary policy to be temporary, for consumption smoothing, they will allocate the increment in their nominal balance between current consumption and savings. As a result, the increase in prices will be less compared with the increase in money supply growth, suggesting that the real money balance increases for active traders and decreases for non-traders.

Overall, the immediate effect of a central bank's unexpected money injection leads to lower interest rates because active traders' savings tend to increase liquidity in the bond market. Segmented market models are thus able to reconcile the liquidity effect with the quantity theory of money. The basic equation derived in Alvarez et al. (2001) study is presented as follows:

$$r_t = \rho + \varnothing\left[E_t\left(\mu_{t+1}\right) - \mu_t\right] + \pi^e \tag{1}$$

where r_t and ρ represent nominal and real interest rate at time t, and π^e is the expected inflation; μ_t represents the growth rate of money supply, which is deemed to be equivalent to the inflation rate in the quantity theory. Let μ_t be constant where $E_t\left(\mu_{t+1}\right) = \mu_t$, in which case agents expect ongoing monetary easing. Equation (1) collapses to the standard fisher equation, $r_t = \rho + \pi^e$, which characterizes long run average money growth, inflation, and interest rates.

Furthermore, observe from equation (2) below, $\varnothing = 0$ when $\lambda = 1$ and $\varnothing$ increases when participation rate drops. Obviously, interest rates will be meaningless if $\lambda = 0$, in which case nobody participates in the bond market.

$$\varnothing = F\left[(1-\lambda)/\lambda\right] \tag{2}$$

From equation (1) the immediate effect of an open market bond purchase can be simplified; when $\mu_t > 0$, interest rate is reduced by $\varnothing\left[\mu_t\right]$. This is the liquidity effect that the segmented market models are designed to capture in general.

The literature has considered many variations of the basic segmented market monetary model, in which economic agents can be differentiated by their connectivity to monetary policy, demonstrating that monetary policy has real effects through distributional considerations (e.g., Azariadis et al., 2019; Zervou, 2013). Research on segmented markets with an emphasis on connections between monetary policy and financial asset pricing are also emerging. For instance, Carlstrom et al. (2017) use such models to show that monetary policy's reaction function should be responsive to the term premium. Peng & Zervou (2022) suggest that central

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bank objectives can have important asset risk implications within the segmented market monetary model.

In our model, the augmentation of the policy risk factor creates a novel thinking process, by which the central bank can influence the effectiveness of its standard policy. When the central bank heightens interest rate uncertainty by frequently performing large-scale open market operations, the utilities of risk-averse traders decrease for any given expected rate of returns, which tends to lower the financial market participation rate. Thereby, a causal relationship is theocratically established between levels of policy effectiveness and the uncertainties generated by policy shocks.

The model incorporates the liquidity effect dynamics in Section II, and the liquidity effect's magnitude from policy shifts is influenced by the monetary policy uncertainties. Sections III and IV introduce the risk of credit default and the sticky information argument into the model, providing more potential links between the participation rate and the policy effects. Section V discusses the notion of an optimal level of policy ambiguity. Sections VI concludes the study.

This model developed for analyzing monetary policy risk could be valuable and relevant to the design of policy framework, as central banks around the globe aggressively change their policy stance, presenting some practical aspects which can be further explored within our theoretical model.

II. Incorporating Variance into the Segmented Market Model

Consider the following model:

$$r_t = \rho + \varnothing\left[\sigma_{\varnothing}\right]\left(E_t\left(\mu_{t+1}\right) - \mu_t\right) + \pi^e \tag{3}$$

The augmented $\sigma_{\varnothing}$ is the standard deviation of $\varnothing$ for a given time interval, servings as a proxy for the uncertainties in monetary policy shocks; $\varnothing$ can be either positive or negative, accounting for expansionary or contractionary monetary policy respectively. Assume that the central bank can control $\sigma_{\varnothing}$ by altering the magnitude and frequency of open market operations. Let's define:

$$\sigma_{\varnothing}^2 = \int f\left(\widehat{\varnothing}\right)\left(\widehat{\varnothing} - \alpha\right)^2 \widehat{\varnothing} d \tag{4}$$

$$\alpha = \int f\left(\widehat{\varnothing}\right)\left(\widehat{\varnothing}\right)\widehat{\varnothing} d \tag{5}$$

For the single variate $\varnothing$, having a probability density function $f\left(\widehat{\varnothing}\right)$ with population mean α, which is equal to the expected value of $\varnothing$ on average, in any given time interval, the variance is defined in equation (4). The integrals are definite integrals taken for $\widehat{\varnothing}$ over the range of $\varnothing$. In the segmented market model incorporating policy risks, $\varnothing$ is deemed to be a continuous random variable.

Equation (3) modifies the standard segmented market model to show that the liquidity effect is amplified by higher levels of variance in monetary policy shocks. Assuming agents are risk averse on average, and lower utilities from trading tend to decrease agents' willingness to participate in the financial market, an increase in the variance of policy shocks $\sigma_{\varnothing}^2$ lowers the market participation rate λ. The Baumol-Tobin model is similarly built to explain portfolio diversification. For a given rate of return, an increase in $\sigma_{\varnothing}^2$ reduces the utility for traders. Thus, fewer agents would be willing to hold bonds. In contrast, λ would be relatively high during when $\sigma_{\varnothing}^2$ is low.

$$\lambda = \lambda(\sigma_{\varnothing}^2) \tag{6}$$

With a lower market participation rate, for any quantity of open market purchase or sale, the changes in the real money balances for a trader becomes greater. The result is greater changes in nominal interest rates, as a repercussion for open market operations. In other words, when λ decreases, nominal interest rates become more responsive to changes in the growth rate of money supply, or open market operations become more effective as a policy tool to control the cost of borrowing.

$$\left|\varnothing_{\mu_t}\right| = \left|\varnothing_{\mu_t}\right|(\lambda) \tag{7}$$

Combining equation (6) and (7), a positive relationship can be drawn from the variance of monetary policy shocks, $\sigma_{\varnothing}^2$ and $\left|\varnothing_{\mu_t}\right|$, which measures the effectiveness of monetary policy in controlling nominal rates.

$$\left|\varnothing_{\mu_t}\right| = \left|\varnothing_{\mu_t}\right|(\sigma_{\varnothing}^2) \tag{8}$$

To sum up, a higher variance of monetary policy shocks lowers the market participation rate, which amplifies the liquidity effect, making the monetary policy more effective.

III. Incorporating Default Risk into the Segmented Market Model

Consider introducing credit risk into the model:

$$r_t = \rho + \varnothing\left[\sigma_{\varnothing}, \tau, \sigma_{\tau}\right]\left(E_t\left(\mu_{t+1}\right) - \mu_t\right) + \pi^e \tag{9}$$

A way to incorporate default risk into the bond market is to incorporate the probability of default τ and its variance σ_{τ}^2 on coupon and principal payments.

$$\tau_t = \vartheta\tau_{t-1} + \widehat{\tau}_t \tag{10}$$

where $0 < \vartheta < 1$, $0 \leq \tau_t$ and $\widehat{\tau}_t$ is an i.i.d. random variable with mean $\aleph > 0$ and variance σ_{τ}^2; $\left(\widehat{\tau}_t - \aleph\right)$ would be positive or negative and $E\left(\widehat{\tau} - \aleph\right)_t = 0$.

$$\begin{aligned} PV = & \sum_{t=1}^{n} C_t\left(1-\tau_t\right)(1+r)^{-t} \\ & + FV_n\left(1-\tau_n\right)(1+r)^{-n} \end{aligned} \tag{11}$$

From equation (11), when the default risk τ of bond increases, the expected rate of return for existing bond holders will decrease to hold the present value constant, the implication being that fewer agents will be willing to participate in the bond market with higher default risks. Furthermore, at any given level of τ, a surge in the variance of default risk σ_{τ}^2 leads to a reduction in the participation rate.

$$\lambda = \lambda(\tau, \sigma_{\tau}^2) \tag{12}$$

Combining equation (7), (8) and (12), its clear that $\sigma_{\varnothing}^2$, τ, and σ_{τ}^2 are all positively related to the effectiveness of monetary policy.

$$\left|\varnothing_{\mu_t}\right| = \left|\varnothing_{\mu_t}\right|(\sigma_{\varnothing}^2, \tau, \sigma_{\tau}^2) \tag{13}$$

Uncertain future returns discourage traders from holding bonds for a given expected return, since they are averse to risk. In addition, the expected bond yield drops when default risk rises, which hurts bond holders' utilities.

IV. The Sticky Information argument in Risk Augmented Model

Agents are mostly unaware of their relative real money balances. Meanwhile, certain cost barriers exist for learning such unobservable information (i.e., the cost of thinking and opportunity cost of time devoted for research and analysis). In each period, assume that only a fraction ψ of the active traders update themselves on the current state of the economy and make asset allocation decisions in real-time. All other traders continue to make portfolio decisions based on outdated information.

In a simplified model scheme, every agent sets portfolio decisions for every period in the bond market, although agents gather information on average price level and calculate their real money balances gradually over time, and the adjustment frequency depends on the perceived uncertainties caused by monetary policy shocks. The underlying assumption regarding information arrival is analogous to the 'adjustment assumption' (Mankiw & Reis, 2002) that each trader has the same probability of being one of the traders updating their decisions, regardless of how long it has been since its last update.

$$\psi = \psi(\sigma_{\varnothing}^2) \tag{14}$$

In equation (14), ψ is positively associated with $\sigma_{\varnothing}^2$, as trading behavior varies with higher uncertainties represented by the variance of policy shocks or greater changes in the short-term rates. Most people spend little time routinely thinking about monetary policies, but circumstances may motivate them to alter the allocation of their mental resources. In other words, how much a person thinks about an issue depends on the benefit of doing so. During a period when large scale open market operations occur frequently, agents would update themselves more, as they discern higher potential payoffs for obtaining timely information on their real money balances.

$$r_t = \rho + \varnothing\left[\sigma_{\varnothing}, \tau, \sigma_\tau, \psi\right]\left(E_t\left(\mu_{t+1}\right) - \mu_t\right) + \pi^e \tag{15}$$

$$\left|\varnothing_{\mu_t}\right| = \left|\varnothing_{\mu_t}\right|\left(\sigma_{\varnothing}^2, \tau, \sigma_\tau^2, \psi\right) \tag{16}$$

When the variance of monetary policy shocks is low, agents know the changes on their relative real money balances are small, with higher λ and lower $\left|\varnothing_{\mu_t}\right|$. Thus, the benefit of updating information and re-evaluating investment decisions tends to be low. When most traders do not immediately realize the underlying effect of open market operations on their relative real money balances, monetary policy shocks could have their maximum impact on the interest rates with substantial delays. In contrast, during periods with extreme policy risks, agents update information more frequently, enhancing the policy's effectiveness.

V. Optimal Risk Considerations

For all the factors in equation (16) link to the effectiveness of monetary policy, only $\sigma_{\varnothing}^2$ is under direct control of the central bank. According to equation (6) and (14), changes in $\sigma_{\varnothing}^2$ affect λ and ψ in opposite directions, thereby indirectly reinforcing the inferred positive relationship between $\sigma_{\varnothing}^2$ and $\left|\varnothing_{\mu_t}\right|$.

Assume ε exists as the threshold level of policy risk above which an unexpected increase in money supply is sufficient to alter the market interest rates. Equivalently, assume λ equals 1 when $\sigma_{\varnothing}$ is sufficiently small, or during periods when $\sigma_{\varnothing} < \varepsilon$, an unexpected change in money supply will not trigger the liquidity effect. In mathematical expressions, this can be presented as:

$\forall\varepsilon > 0 \exists \sigma_{\varnothing} < \varepsilon$ such that $\varnothing = 0$

$\forall\varepsilon > 0 \exists \sigma_{\varnothing} \geq \varepsilon$ such that $\varnothing = \varnothing\left[\sigma_{\varnothing}\right]\left(E_t\left(\mu_{t+1}\right) - \mu_t\right)$

Define equation (17) as follows:

$$\varnothing\left[\sigma_{\varnothing}\right] = \begin{cases} 0, \sigma_{\varnothing} < \varepsilon \\ \varnothing\left(\dfrac{\sigma_{\varnothing}}{\sigma_{\varnothing} + 1}\right), \sigma_{\varnothing} \geq \varepsilon \end{cases} \tag{17}$$

When default risk is considered, monetary policies become more effective for controlling interest rates, so long as τ and σ_τ become sufficiently large, even when endogenous policy risks have been low $\sigma_{\varnothing} < \varepsilon$.

Credit risks tend to be high during economic downturns, so our model implies that the policy is generally more effective in recessions than in booms. By analogy, as default risks can differ significantly among different market economies, it can be inferred that the policy tends to be more effective in the economies suffering relatively high levels of credit risks. Overall, the uncertainties in macroeconomic environment can be a key factor for policy effectiveness.

VI. Conclusion

Once random shocks occur, near term losses would be inevitable despite the central bank's reactions. These disturbances can render original optimization levels temporarily unachievable. Uncertainties regarding expected future returns affect the market participation rate, which is linked with the effectiveness of open market operations for controlling interest rates.

The analytical framework presents a new dimension for consideration when setting the optimal interest-rate controlling strategy. For instance, when the default risks of several EU members sharply increased during the Eurozone debt crisis, the European Central Bank calmed financial markets by large scale expansion of the monetary base. Generally, our model can be adopted to further explore optimal policy reactions to economic crisis.

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Appendix

Instead of assuming a fraction of agents never participate in the bond market, consider a more realistic assumption that agents tend to put different proportional weights on bonds in their asset portfolios. Expansionary monetary policy then increases the real money balance of agents holding relatively more bonds.

Let's assume:

- Agents live for two periods (0, 1)
- Agents derive utility from consumption C
- $U_c > 0 \; U_{CC} < 0$
- Each agent born with the same initial endowments b
- Received income y_0 which is known and y_1 which is in expectation
- Discount future utility level with $\beta(\tau)$ which depends on the probability of not being able to receive the expected utility from consumption in period 1

$$Max\, U(C_0) + \beta E[U(C_1)] \qquad 0 < \beta \leq 1 \tag{A.1}$$

Subject to $C_0 + \frac{C_1}{1+\rho} = b + y_0 + \frac{E(y_1)}{1+\rho}$

$$\begin{aligned} L &= U(C_0) + \beta E[U(C_1)] \\ &+ \delta(b + y_0 + \frac{E(y_1)}{1+\rho} - C_0 - \frac{C_1}{1+\rho}) \end{aligned} \tag{A.2}$$

$$\frac{\partial L}{\partial C_0} = U'(C_0) - \delta = 0$$

$$\frac{\partial L}{\partial C_1} = \beta U'(C_1) - \frac{\delta}{1+\rho} = 0$$

$$U'(C_0) = \beta(1+\rho)E\left[U'(C_1)\right] \tag{A.3}$$

The general form of equation (A.3) characterizes the consumption plan over time:

$$U'(C_t) = \beta(1+\rho)E\left[U'(C_{t+1})\right] \tag{A.4}$$

As β is the discount factor considering agents' degree of risk aversion on the risk of default, and $(1+\rho)$ is the returns for saving one period, then ρ includes risk premium which is the part of return on bearing the risk of default and the variance on expected return. When $\beta(1+\rho) = 1$, the effects on these two factors offset each other exactly.

$$\text{If } \beta(1+\rho) = 1 \text{ then } U'(C_t) = E\left[U'(C_1)\right] \quad C_t = E(C_1)$$

$$\text{If } \beta(1+\rho) > 1 \text{ then } U'(C_t) > E\left[U'(C_1)\right] \quad C_t < E(C_1)$$

$$\text{If } \beta(1+\rho) < 1 \text{ then } U'(C_t) < E\left[U'(C_1)\right] \quad C_t > E(C_1)$$

Assuming the degree of risk aversion β differs among agents, agents who are less risk averse have higher β. Less risk averse agents tend to consume more in the future according to the condition set by equation (A.4). Holding everything else constant, one must save more in time t to consume more in time t+1. Thus, less conservative agents with higher β are likely to hold a larger asset portfolio and to be more active in the financial market. So long as agents put different weights on bond holdings, open market operations will create discrepancies in agents' real money balances, and thus the liquidity effect in the incomplete market model holds.